HOMOTHERMAL SHOCKWAVE PROPAGATION IN AN ATMOSPHERE NEAR A COMPACT MEDIUM

L. P. Gorbachev and V. F. Fedorov UDC 536.45:533.6.011

Problems describing gas motion with energy liberation near the boundary of two media of inhomogeneous densities are examined in a number of papers (see, e.g., [1-5]). The influence of thermal radiation of the medium on the regularities of its motion becomes substantial for a high density of the energy being liberated [5, 6].

Within the framework of a homothermal model, the regularities of gas motion in an atmosphere are studied with evaporation of the condensed phase taken into account.

In a condensed medium such as water, let the radiant energy E_0 be liberated instantaneously near its surface in an infinitesimal volume. For a high radiation energy density, the temperature in the whole perturbation domain is equalized instantaneously because of intensive heat transfer, and a homothermal approximation can be utilized to estimate the characteristics of the occurring gas motion. We assume that the temperature depends only on the time $T(r, t) \equiv T(t)$.

A thin surface layer of the compact substance is heated and evaporated owing to absorption of the thermal radiation. The pressure gradient originating on the vapor—air interface results in motion of the substance being evaported in the air medium. A shockwave is propagated from the energy liberation site into the air. The density diminution due to the vapor motion contributes to radiation heating of the subsequent condensed medium layers and displacement of the vapor-compact medium interface.

In this case, the fraction of the mechanical energy transmitted to the compact medium is negligible. According to [2], 2% of the energy being liberated during an explosion on a soil surface is transmitted to the soil in the form of mechanical energy due to diffusion of the radiation. Therefore, the shock being propagated in the compact medium can be neglected, and the compact medium can be considered nondeformable.

Therefore, a rarefaction wave whose leading front is the vapor-compact medium interface is propagated in a condensed medium at the isothermal speed of sound from the site of an explosion. We call such a rarefaction wave radiational.

We simulate the air and vapor by an ideal gas with the effective parameters γ_i, μ_i ($i = 1$ for air and $i = 2$ for the vapor). We neglect losses in the energy E_0 being liberated due to evaporation, dissociation, and ionization of the medium.

The equations describing the homothermal gas motion (vapor and air) in the perturbation domain have the form

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{u}) = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\,\mathbf{u} + a^2 \nabla(\ln \rho) = 0, \tag{1}$$

where $a = \sqrt{AT}$ is the isothermal speed of sound, and A is the gas constant. The gas flow is axisymmetric. Two velocity and gas density components must be determined as functions of two space coordinates and the time.

The problem can be considered in a self-similar formulation. The system of defining parameters in this problem is as follows: r, z (if a cylindrical coordinate system is utilized), E_0, ρ_1, ρ_2, γ_1, γ_2, μ_1, μ_2, (ρ_1, ρ_2 are the densities of the unperturbed air and vapor). All the dimensionless gas flow characteristics can be considered as functions of the dimensionless coordinates $r(\rho_1/E_0)^{1/5}t^{-2/5}$, $z(\rho_1/E_0)^{1/5}t^{-2/5}$ and the constants ρ_2/ρ_1, γ_1, γ_2, μ_1, μ_2. However, even the self-similar problem turns out to be complex since it is two-dimensional. Equations (1) do not reduce to a system of ordinary differential equations.

The situation is made even more complicated by the fact that the shock front and rarefaction wave surfaces on which the boundary conditions are given in addition to the system

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 70-73, January-February, 1988. Original article submitted October 9, 1986.

0021-8944/88/2901-0065$12.50

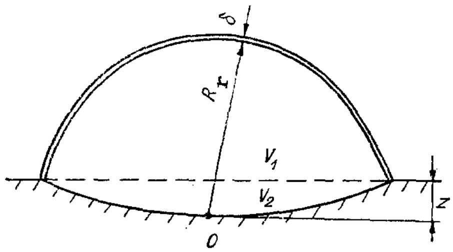

Fig. 1

TABLE 1

Crater shape	ξ_0	ξ_1, $m^{-2}\cdot sec^{-1}\cdot K$	β
Cone	1,18	$4{,}59\cdot10^{-6}$	9,68
Segment	1,175	$3{,}68\cdot10^{-6}$	10,93
Hemispheroid	1,167	$2{,}97\cdot10^{-6}$	11,9

(1) are not defined; they should be found during the solution. For this reason, even numerical integration of (1) in the case of self-similar motion is fraught with significant difficulties.

Let us consider the approximate solution of the problem formulated on the basis of using the integral mass, momentum, and energy conservation laws [7, 8]. It is shown in [8] that this method assures acceptable accuracy in many cases.

We assume that the shock front velocity D is much greater than the speed of sound in the heated gas a ($\beta = D/a \gg 1$). The air shock collects the air mass it perturbed in a thin layer δ near the front. We assume the thickness of the layer δ infinitely small, while we take the density in the layer almost infinite.

In this case the integral gas momentum, energy, and mass conservation laws can be written in the form

$$d(M_1D)/dt = S_r p_v; \tag{2}$$

$$E_0 = M_1(D^2/2 + a^2\mu_2/\mu_1/(\gamma_1 - 1)) + M_2a^2(\varepsilon_K + 1/(\gamma_2 - 1)); \tag{3}$$

$$M_1 = \rho_1V_1,\ M_2 = \rho_2V_2. \tag{4}$$

Here $p_v = a^2\rho_v$ is the vapor pressure behind the shock front, $\rho_v = \rho_2V_2/V$ is the vapor density in the cavity, M_1 and M_2 are the air and vapor masses included in the motion, $S_r = 2\pi R_r(R_r - Z)$ is the area of the shock front surface, which we assume to be spherical with center at the point 0, being displaced in the compact medium according to the law $Z = \int_0^t a(t')dt'$ (see Fig. 1). $V = V_1 + V_2$ is the volume of the perturbed medium, $V_1 = \pi(R_r - Z)^2(2R_r + Z)/3$ is the volume of the spherical segment, and V_2 is the volume of the crater. $\varepsilon_K = \dfrac{E_{sc}}{M_2a^2} = \dfrac{1}{M_2a^2}\int_V \dfrac{\rho u^2}{2}dV$ is the dimensionless kinetic energy of the vapor.

To make estimates according to (2)-(4), it is necessary to know the gas density and velocity distributions behind the rarefaction wave front and the shape of its front. We take into account that heating and evaporation at this point of the compact medium surface start from the time when the air shock reaches it, and that the vapor motion in a direction perpendicular to the interfacial surface of the media will be predominant.

We turn now to the results of solving the simplest problem for a plane geometry while neglecting air density [9]. Propagation of a radiational rarefaction wave from the interface of a condensed medium-vacuum is described by analytical dependences in the isothermal case. In particular, the vapor velocity and density distributions satisfy the relationships $u_z = a(1 + z/at)$, $\rho = \rho_2\exp(-1 - z/at)$, where the z axis is perpendicular to the interfacial surface of the media; $-at < z < \infty$; $t > 0$.

Note that even in the homothermal approximation, the problem about the regularities of rarefaction wave propagation from the interface of a condensed medium—vacuum interface under the instantaneous liberation of the energy E_0 per unit surface will result in analogous dependences: the velocity grows with distance from the wave front according to a linear law, while the density decreases exponentially [9]. Taking these results into account, the values of the density ρ_V and the dimensionless kinetic energy ε_K of the vapor are determined from the formulas

$$\rho_V = \frac{\rho_2 V_2}{V} = \frac{\rho_2}{\beta}\int_0^\beta \exp(-\alpha\xi)\,d\xi; \tag{5}$$

$$\varepsilon_K = \frac{E\,\mathrm{kv}}{M_2 a^2} = \frac{V}{V_2}\frac{1}{\beta}\int_0^\beta \frac{\exp(-\alpha\xi)}{2}\,\xi^2 d\xi \tag{6}$$

(ξ is a dimensionless coordinate).

Taking account of (2) and (4), we obtain from the energy balance equation (3) a time dependence of the shock front radius which can be reduced to the "standard" form

$$R_r = \xi_0 (E_0/\rho_1)^{0.2} t^{0.4}, \tag{7}$$

$$\xi_0 = \beta \Big/ \left\{\frac{4\pi}{25}\left[\left(\frac{\beta^2}{2} + \frac{\mu_2}{\mu_1(\gamma_1 - 1)}\right)\frac{(\beta-1)^2(2\beta+1)}{3} + \frac{\rho_2}{\rho_1}\left(\varepsilon_K + \frac{1}{\gamma_2 - 1}\right)\frac{V_2}{z^3}\right\}^{0.2}.$$

The gas temperature is calculated from the formula $T = (dR_r/dt)^2/A\beta^2$ and can be converted by taking account of (7) into

$$T = \xi_1 (E_0/\rho_1)^{0.4} t^{-1.2}, \quad \xi_1 = (\xi_0/\beta)^2 \mu_2/R. \tag{8}$$

Assume given the following values of the initial parameters for the estimates: $\rho_1 = 1.29$ kg/m^3, $\gamma_1 = 1.37$, $\mu_1 = 2.02$ [8], $\rho_2 = 10^3$ kg/m^3, $\gamma_2 = 1.52$, $\mu_2 = 2.57$ [10]. Let us examine different possible crater forms: a cone ($V_2 = \pi z^3(\beta^2 - 1)/3$), a spherical segment ($V_2 = \pi z^3(3\beta^2 - 2)/6$), a hemispheroid ($V_2 = \pi z^3 2(\beta^2 - 1)/3$). We determined β from (2) for each case, α from (5), and ε_K, from (6), which permits computation of ξ_0 and ξ_1, where the values of the desired parameters are presented in Table 1. A sufficiently weak dependence of the desired parameters on the choice of the crater geometry is detected and the approximate solution constructed can be considered acceptable for rough estimates.

For comparison we present analogous data without taking the evaporation of the compact medium into account. The solution of the self-similar problem in a homothermal approximation [5] results in dependences analogous to (7) and (8) but with different values of ξ_0 and ξ_1. For the selected values of ρ_1, γ_1, and μ_1 and the energy $2E_0$ of the explosion, $\xi_0 = 1.29$ and $\xi_1 = 1.57 \cdot 10^{-5}$ sec$^2 \cdot$m$^{-2} \cdot$K.

Therefore, taking account of the mass of vapor involved in motion results in a moderate diminution of the shock front radius and a significant diminution in the gas temperature.

LITERATURE CITED

1. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shocks and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).
2. V. N. Nikolaevskii (ed.), Shock, Explosion, and Rupture [in Russian], Mir, Moscow (1981).
3. S. S. Grigoryan and L. S. Evterev, "On the effect of a strong explosion on the surface of a cleavage half-space," Dokl. Akad. Nauk SSSR, 222, No. 3 (1975).
4. L. V. Shurshalov, "On the analysis of a shock being propagated in an inhomogeneous atmosphere," Dokl. Akad. Nauk SSSR, 230, No. 4 (1976).
5. V. P. Korobeinikov, Problems of the Theory of a Point Explosion in Gases [in Russian], Nauka, Moscow (1985).
6. V. F. Fedorov, "On a homothermal shock caused by the action of instantaneous monochromatic radiation," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1979).
7. G. G. Chernyi, Gas Flow at High Supersonic Velocity [in Russian], Fizmatgiz, Moscow (1959).
8. L. P. Gorbachev and V. F. Fedorov, "On the influence of the mass being liberated during an explosion on thermal wave propagation," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1978).
9. V. F. Fedorov, "On homothermal gas motion near a compact medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1987).
10. V. N. Nikolaevskii (ed.), Underwater and Underground Explosions [in Russian], Mir, Moscow (1974).